Anomaly Detection based on Markov Data: A Statistical Depth Approach

Carlos A. Fernández[1] and Stephan Clémençon[1]

[1] *LTCI, Télécom Paris, Institut Polytechnique de Paris, France*

Abstract

The purpose of this article is to extend the notion of statistical depth to the case of sample paths of a Markov chain. Initially introduced to define a center-outward ordering of points in the support of a multivariate distribution, depth functions permit to generalize the notions of quantiles and (signed) ranks for observations in $\mathbb{R}^d$ with $d > 1$, as well as statistical procedures based on such quantities. Here we develop a general theoretical framework for evaluating the depth of a Markov sample path and recovering it statistically from an estimate of its transition probability with (non-) asymptotic guarantees. We also detail some of its applications, focusing particularly on unsupervised anomaly detection. Beyond the theoretical analysis carried out, numerical experiments are displayed, providing empirical evidence of the relevance of the novel concept we introduce here to quantify the degree of abnormality of Markov paths of variable length.

Keywords: statistical depth functions, anomaly detection, non-parametric statistic, Markov chains

References

Carlos A. Fernández and Stephan Clémençon. Anomaly detection based on markov data: A statistical depth approach. Submitted to LOD 2025, 2025. URL https://arxiv.org/abs/2406.16759.