## Non-linear monotone models for general Harris recurrent Markov chains

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## Abstract

In this paper, we study a nonlinear cointegration-type model of the form  $Z_t = f_0(X_t) + W_t$ where  $f_0$  is a continuous monotone function and  $X_t$  is a Harris recurrent Markov chain, which may be either positive or  $\beta$ -null recurrent. We use a nonparametric Least Square Estimator based on the concave majorant to locally estimate  $f_0$ . Under mild conditions, we show its strong consistency and obtain its rate of convergence. In the positive recurrent case, the rate is similar to the optimal one obtained in the i.i.d case that is of order  $n^{1/3}$ . In the  $\beta$ -recurrent case, the rate of convergence is essentially linked to the local properties of the  $\beta$ -null recurrent chain  $X_t$  and typically of the order  $n^{\beta/3}$ , up to some slowly varying function. The main advantage of our estimator is that we do not need any smoothing parameters, neither in a stationary nor in a non-stationary context and the rate does not depend on any additional smoothness assumptions. New results (of the Glivenko-Cantelli type) for localized null recurrent Markov chains are also proved.

**Keywords:** monotone regression, isotonic regression, nonlinear cointegration, nonparametric estimation, null recurrent Markov chains

## References

Patrice Bertail, Cécile Durot, and Carlos A. Fernández. Harris Recurrent Markov Chains and Nonlinear Monotone Cointegrated Models. 2024. URL https://arxiv.org/abs/2407.05294.