

The calculation of the probability of observations in a HMM knowing the parameters of the model can be done classically with the forward or the forward-backward algorithm. Knowing that, we formalize the notion of an un-normalized heterogeneous Markov distributions (UHMD), as a tensor associated to a joint distribution, where these algorithms correspond to an elimination and marginalisation algorithm respectively, based on a series of matrix x vector products on a tensor network. It leads to a complexity in  $O(TK^2)$  for normalising constant and computing all unary marginals. We show how the sparsity of the transition matrix permits to decrease the complexity of the calculation. We use this dictionary to extend within a common framework the evaluation of the complexity of forward and forward-backward algorithms for a diversity of Markov based Models with hidden variables. We show how to map the joint law of a HMM into a UHMD for computing the distribution of the observations after marginalisation over the hidden variables. This is extended to multichain HMM, and leads to bound the complexity according to the type of couplings using the decomposition or not of the transition matrix as a Kronecker product of univariate matrices, including the Factorial Model. We use the generative model for an ED-HMM to rewrite it as a UHMD, show that the transition matrix is sparse, and show how the sparsity can lower the upper bound of the complexity of the forward-backward algorithm. We present ways to possibly extend this approach towards some cases of multichains HSMM. Finally, we recall that the field property of  $\mathbf{R}$ , i.e. the inversion of the multiplication, and even of the addition, is not required in matrix x vector product in the definition of an UHMD. This means that all these calculations can be implemented in a semi-ring. Such an implementation is classical in max-plus semi-ring and leads to Viterbi algorithm, corresponding to an elimination algorithm, which can be deployed this way on all mentioned models of the Markov family to recover most likely hidden states.