# Piecewise Deterministic Markov Processes and Bacterial Growth

*Bertrand Cloez, Benoîte de Saporta , **Nathalie Krell** ,Tristan Roget, Emeline Schmisser*

**Abstract**

This presentation will focus on the study of piecewise deterministic Markov process (PDMP) models used to describe dynamic phenomena involving occasional random events. In the first part, I will present work done in collaboration with Emeline Schmisser [6] on the nonparametric estimation of the jump rate of a deterministic PDMP based on observations of the embedded Markov chain $(Z_k)$. We construct an adaptive estimator of the jump rate $\lambda$ and establish an $L^2$ norm risk bound as well as a quasi-minimax result.

In a second part, I will focus on a bacterial growth model based on a PDMP-type multi-type branching process. Each cell grows exponentially at an individual rate, then divides at a rate depending on its size. This work extends a model introduced by Doumic, Hoffmann, Krell, and Robert [2] by integrating two cell types according to pole position. We show that the process is well defined, satisfies a "many-to-one" formula, and that its average empirical measure converges to a growth-fragmentation equation with multiple state variables. I will conclude with perspectives on estimating the division rate in this framework, in collaboration with Benoîte de Saporta, Bertrand Cloez, and Tristan Roget [1]. To this end, I will rely on a proceedings paper [4] in which I constructed a two-type branching process to model bacterial growth.

## References

[1] B. Cloez, B. de Saporta, N. Krell and T. Roger. *Investigation of asymmetry in E. coli growth rate.* Work in progress.

[2] Doumic, M., Homann.,M., Krell, N. and Robert, L. (2015) *Statistical inference across scales for size-strutured models under growth variability.* Bernoulli, 21, 1760-1799.

[3] Doumic, M., Homann.,M., Krell, N., Robert, L., Aymerich S. and Robert J. (2014) *Division Control in Escherichia coli is Based on a Size-sensing rather than Timing Mechanism.* BMC Biology 12:17, 2014.

[4] N. Krell 2025 *Branching processes and bacterial growth.* To appear Proceedings IWBPA24.

[5] N. Krell (2016) *Statistical estimation of the jump rate of a special class of PDMPs. ESAIM: PS 20 196–216.*

[6] N. Krell , and E. Schmisser (2021) *Nonparametric estimation of jump rates for a specific class of piecewise deterministic Markov processes. Bernoulli, 27(4):2362–2388.*