The modeling of epidemic dynamics is essential in the field of epidemiology, as new methods offer better estimates of disease spread. Deterministic approaches often fail to fully capture epidemic evolution due to the inherent uncertainty in these processes. A shift to a stochastic perspective is therefore necessary, explored here through three fundamental epidemiological models. A stochastic SIRD (Susceptible (S), Infectious (I), Recovered (R), Deceased (D)) model with imperfect immunity is presented, based on a continuous-time Markov chain. Formulas and algorithms are developed to compute probabilities and moments of time-related quantities, such as disease extinction time, alarm time, and the infection time of a susceptible individual. Sensitivity analysis is then used to examine the behavior of these quantities with respect to the parameters of the stochastic model. A Markovian SIHRD (Susceptible (S), Infectious (I), Hospitalized (H), Recovered (R), Deceased (D)) model is also introduced to analyze hospitalizations, focusing on size-related characteristics like total and maximum hospitalizations, and their joint distribution with infections. These analytical tools are then combined with dynamic parameter estimation via particle filtering, using a SPIR (Susceptible (S), Presymptomatic (P), Infectious (I), Recovered-Deceased (R)) model to capture time-varying epidemic behavior. This includes tracking the number of infections until extinction, the timing of specific death counts, and infections caused by presymptomatic or infectious individuals. Applied to monkeypox (mpox) data from Ghana, the method produces more accurate estimates than fixed-parameter models. Sensitivity analysis further reveals how infectious and presymptomatic individuals influence transmission.