Multichain Hidden Markov Models

Nathalie Peyrard
INRAE, Unité MIA, Toulouse

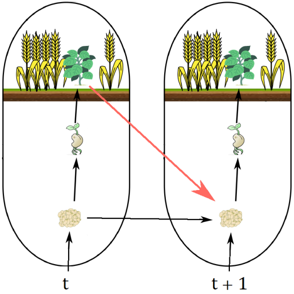
with the 'multichain' working group of ANR project HSMM-INCA
Hanna Bacave, JB Durand, Alain Franc, Sandra Plancade, Régis Sabbadin

MaSeMo workshop - July 2, 2025

Outline of the presentation

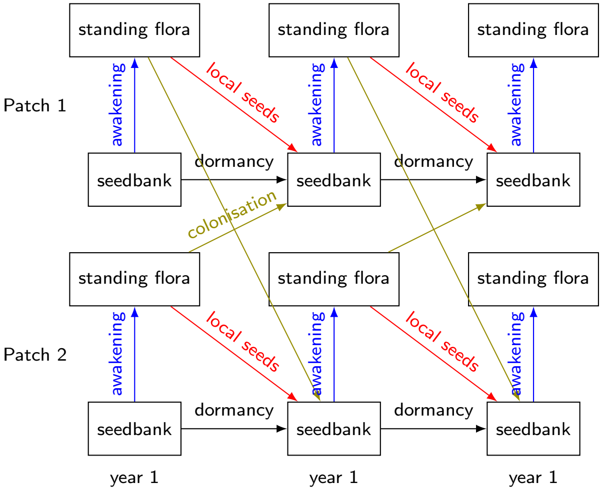
Annual plants dynamics: one patch

- How can we estimate the key parameters of annual plant dynamics based only on observations on standing flora?



- Dormancy: seed survival
- Seedbank: **not visible**
- Dynamics: germination, seed production, dormancy and colonisation

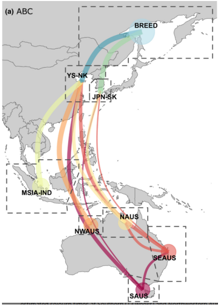
Annual plants dynamics: several patches



[with S. Le Coz at MIAT and P.-O. Cheptou at CEFE Montpellier]

Birds migration routes

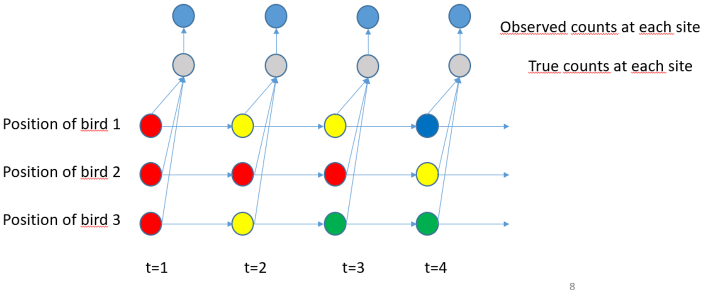
- Which are the main routes used by the shorebirds?



- several stop overs
- possible routes form a network
- bird trajectories **not available**
- data: imperfect counts (eBird data)

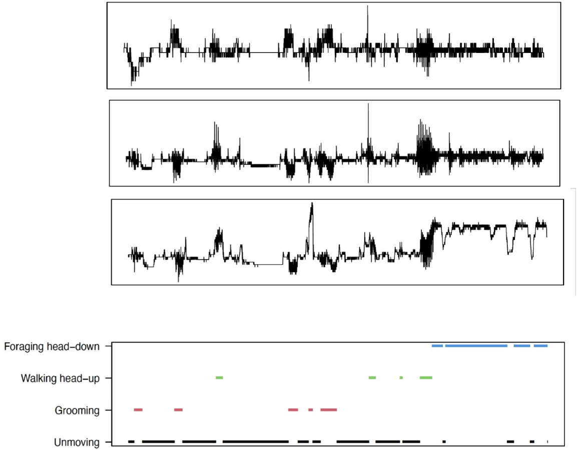
[with R. Sabbadin, R. Trépos and M.-J. Cros at MIAT, and S. Nicol at CSIRO Brisbane]

Birds migration routes



Deers behaviour

- How to reconstruct successive activity phases from accelerometry data?

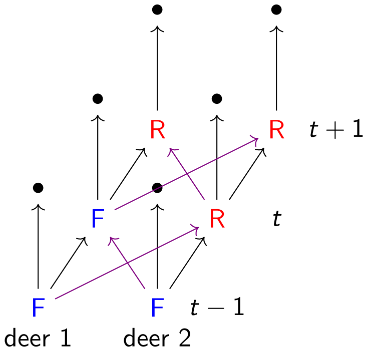


[S. Plancade at MIAT and N. Morellet, N. Ranc at CEFS]

Deers behaviour: several individuals

- Several deers in a same location: change in one animal's activity phase (**hidden**) may impact behaviours of other animals

$$\begin{cases} \mathbb{P}[Z^1_{t+1} = running | Z^1_t = foraging, Z^2_t = running] & = & 0.7 \\ \mathbb{P}[Z^1_{t+1} = running | Z^1_t = foraging, Z^2_t = foraging] & = & 0.1 \end{cases}$$



A common framework for dynamics of temporal processes in interaction with hidden states?

- Common features :
  - several individual dynamics
  - that are not independent
  - with two levels : hidden state (regime, class, ...) and observations (hidden+noise, proxy of hidden, ...)
- Differences : interaction does not always take place at the same level
- Other examples : epidemiology, earthquakes, signal processing...

Natural framework: HMM (already largely used for one individual dynamics)

$$\begin{array}{ccccccccccc}
Y_t & \bullet & & \bullet & & \bullet & & \bullet & & \bullet & & \bullet \\
 & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow \\
Z_t & \circ & \longrightarrow & \circ & - - - - - & \circ & \longrightarrow & \circ & - - - - - & \circ & \longrightarrow & \circ \\
 & 0 & & 1 & & t & & t+1 & & T-1 & & T
\end{array}$$

This talk is about

- reviewing existing HMM based models for modeling interacting temporal processes with hidden states
- proposing a general definition of multichain HMM
- discussing estimation complexity

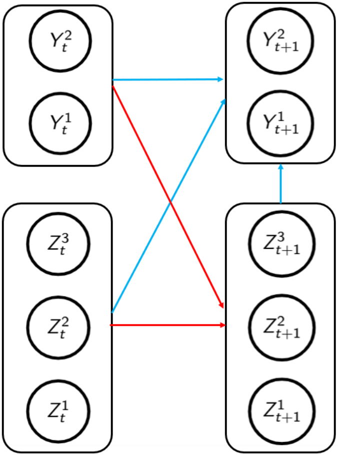
Starting point

Variables:

- Hidden: $\boldsymbol{Z}_t$ with C components (C chains): $\boldsymbol{Z}_t = (Z_t^1, \ldots, Z_t^C)$ with $Z_t^c \in \Omega_{Z^c}$.
- Observed: $\boldsymbol{Y}_t$ with O components (O observations): $\boldsymbol{Y}_t = (Y_t^1, \ldots, Y_t^O)$ with $Y_t^o \in \Omega_{Y^o}$.

Proposition of a hierarchy of definitions built by successively adding conditional independencies assumptions

General MHMM



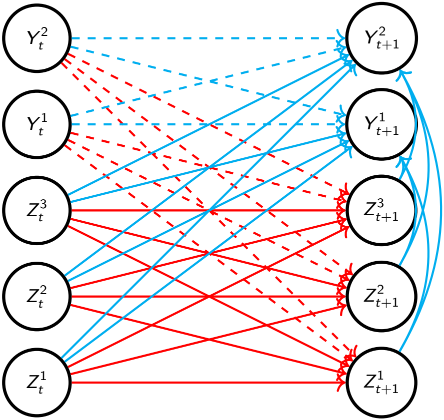
- $(\boldsymbol{Z}_t, \boldsymbol{Y}_t)_{t\in\mathbb{N}}$ is Markovian

$$\begin{aligned}
&\mathbb{P}(\boldsymbol{Z}_{t+1}, \boldsymbol{Y}_{t+1} \mid \boldsymbol{Z}_t, \boldsymbol{Y}_t) \\
&= \mathbb{P}(\boldsymbol{Y}_{t+1} \mid \boldsymbol{Z}_{t+1}, \boldsymbol{Z}_t, \boldsymbol{Y}_t) \times \mathbb{P}(\boldsymbol{Z}_{t+1} \mid \boldsymbol{Z}_t, \boldsymbol{Y}_t) \\
&= \textit{emission} \times \textit{transition}
\end{aligned}$$

- limit: space needed for representation $|\Omega_Z|^{2C} \times |\Omega_Y|^{2O}$

MHMM with conditional independencies in emission and transition

- in emission (resp. transition) terms, additional assumption of conditional independencies of the Y^o_{t+1} (the Z^c_{t+1})
- factorisation
  - $\mathbb{P}(\boldsymbol{Z}_{t+1} \mid \boldsymbol{Z}_t, \boldsymbol{Y}_t) = \prod_{c=1}^{C} \mathbb{P}(Z^c_{t+1} \mid \boldsymbol{Z}_t, \boldsymbol{Y}_t)$
  - $\mathbb{P}(\boldsymbol{Y}_{t+1} \mid \boldsymbol{Z}_{t+1}, \boldsymbol{Z}_t, \boldsymbol{Y}_t) = \prod_{o=1}^{O} \mathbb{P}(Y^o_{t+1} \mid \boldsymbol{Z}_{t+1}, \boldsymbol{Z}_t, \boldsymbol{Y}_t)$



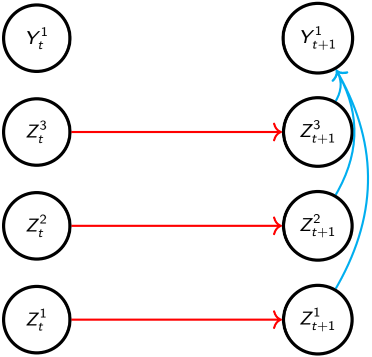
MHMM with conditional independencies in emission and transition

- gain in representation space
  - $C|\Omega_Z|^{C+1} \times |\Omega_Z|^{O}$ terms for transition
  - $O|\Omega_Z|^{2C} \times |\Omega_Z|^{O+1}$ terms for emission
- factorisation meaningful in many applications (weeds, birds, deers)
- some dynamical graphical models from literature are examples MHMM-CI

Subcase 1 of MHMM-CI

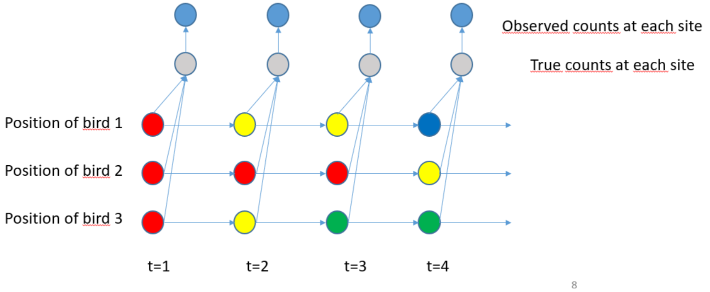
Factorial HMM (FHMM, [Ghahramani and Jordan, 1997])

- $O = 1$
- independent hidden Markov chains
- joint emission of the observation



Subcase 1 of MHMM-CI

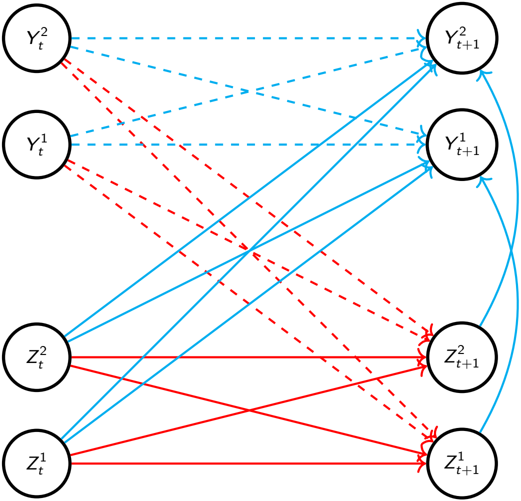
Application: birds migration routes



Subcase 2 of MHMM-CI

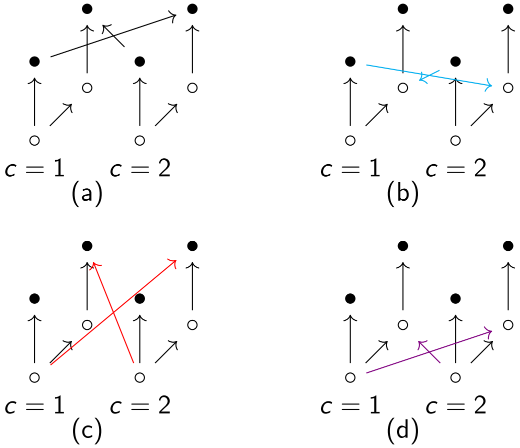
1to1-MHMM-CI: one observation per hidden variable

- $O = C$
- $\mathbb{P}(Y^c_{t+1} \mid \boldsymbol{Z}_{t+1}, \boldsymbol{Z}_t, \boldsymbol{Y}_t) = \mathbb{P}(Y^c_{t+1} \mid Z^c_{t+1}, \boldsymbol{Z}_t, \boldsymbol{Y}_t)$



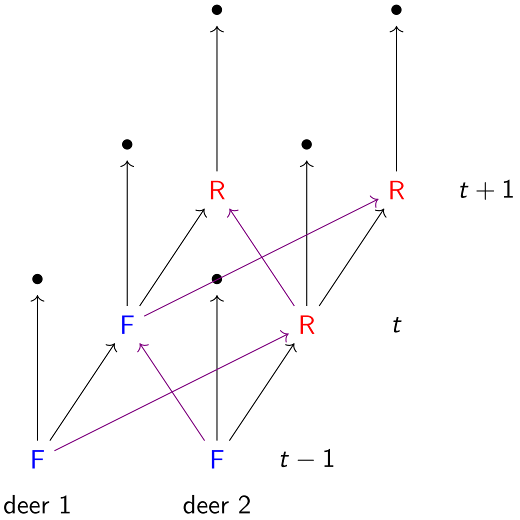
Subcase 2 of MHMM-CI

Building **1to1-MHMM-CI** by coupling HMM structures



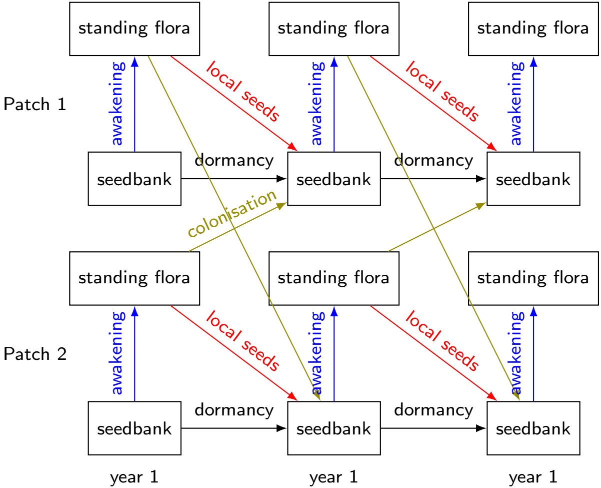
Subcase 2 of MHMM-CI

Application: deers behaviours as a CHMM [Brand, 1997]

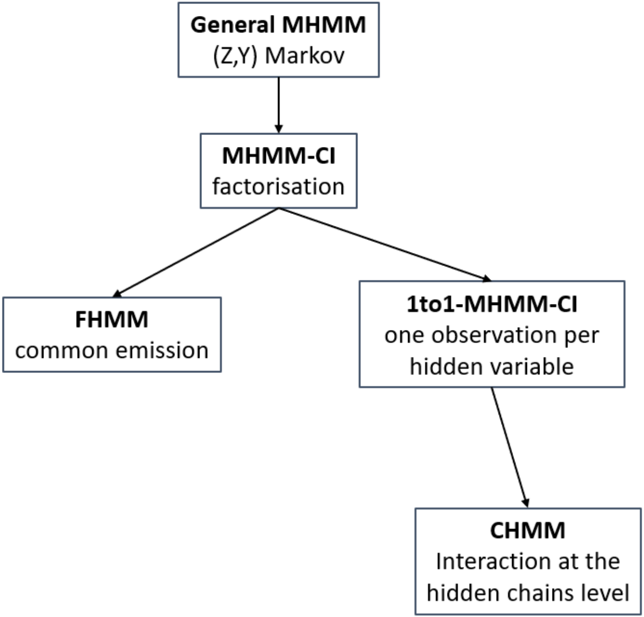


Subcase 2 of MHMM-CI

Application: weeds dynamics [Le Coz et al., 2019]



Typology


General MHMM
(Z,Y) Markov
MHMM-CI
factorisation
FHMM
common emission
1to1-MHMM-CI
one observation per
hidden variable
CHMM
Interaction at the
hidden chains level

Tractability of MHMM estimation

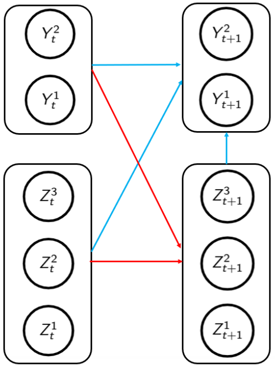
Estimation of the emission and the transition

Context

- EM: classical algorithm for computing the Maximum Likelihood Estimator in models with hidden (latent) variables
- EM for HMM
  - standard tool (many packages)
  - relies on the forward-backward algorithm
  - time complexity $\mathcal{O}(TK^2)$ (T, length of observation sequence, K, number of hidden states)

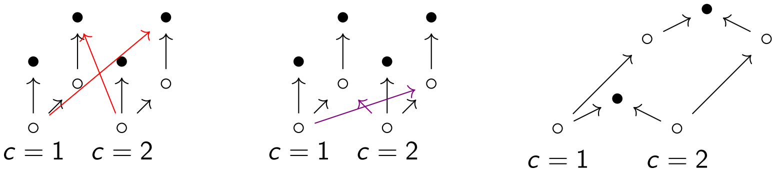
Question: is it still stractable for MHMM? Does it depends on the dependency structure?

Complexity of EM for a General MHMM



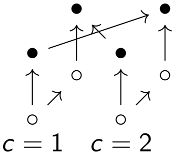
- a MHMM is an HMM with large state space (up to some adaptation)
- state space of $\boldsymbol{Z}_t$ is of dimension K^C
- EM complexity is $\mathcal{O}(TK^{2C})$: **untractable**

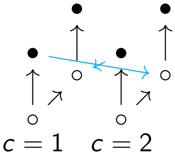
Complexity of EM for some MHMM-CI



- Hidden chains are not independent conditionally to the observations
- The forward-backward recursions must be run on the multidimentional hidden state
- EM complexity is $\mathcal{O}(TK^{2C})$: **untractable**

Complexity of EM for some MHMM-CI





- Hidden chains are independent conditionally to the observations
- A forward-backward algorithm is run independently for each chain
- EM complexity is $\mathcal{O}(CTK^2)$: **tractable**

Motivation

Multichain HMM: a typology

Multichain HMM: tractability of estimation

Conclusion

Conclusion

- A general framework that includes and generalizes well known structures of multichain HMMs (FHMM, CHMM)
- Various dependencies structures = can cover many different applications
- Parameter estimation
  - exact EM in general exponential in number of chains but can be linear for some structures
  - if exponential complexity, classical options for approximate inference are Monte Carlo, variational approximation or simplifying assumptions
- Beyond: multichain HSMM
  - the classical representation by state and duration not adapted anymore
  - we propose a rigorous definition based on hazard rate ... but that would be another talk

References

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